PHYS331: Optics

Equations & Constants

Version 2: December 11, 2020

Constants

- Speed of EM wave in vacuum: $c = 2.998 \times 10^5\,\mathrm{km\,s^{-1}} = 1/\sqrt{\varepsilon_0\mu_0}$
- Permittivity of vacuum: $\varepsilon_0 = 8.854 \times 10^{-12}\,\mathrm{C^2\,N^{-1}\,m^{-2}}$
- Permeability of vac.: $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{T\,m\,A^{-1}}$
- Wien's Displacement Law constant: $\kappa = 2.898 \times 10^6\,\mathrm{nm\,K}$
- Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
- Planck constant: $h = 6.626 \times 10^{-34}\,\mathrm{J\,s}$
- Boltzmann constant: $k_\mathrm{B} = 1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}}$
- Coulomb constant: $k_e = 1/(4\pi\varepsilon_0) = 8.988 \times 10^9\,\mathrm{N\,m^2\,C^{-2}}$
- Electron-volt: $1\,\mathrm{eV} = 1.602 \times 10^{-19}\,\mathrm{J}$
- Tesla: $1\,\mathrm{T} = 1\,\mathrm{kg\,A^{-1}\,s^{-2}}$

Indices of Refraction			
Vacuum	1	Pyrex	1.47
Air	1.000277	Crown glass	1.50
Water	1.3330	Flint glass	1.60
Human eye	1.34	Diamond	2.419

Equations

- Lateral magnification: $m = \dfrac{h'}{h} = -\dfrac{n_1 s'}{n_2 s}$
- Refraction at spherical surface: $\dfrac{n_1}{s} + \dfrac{n_2}{s'} = \dfrac{n_2 - n_1}{R}$
- Lensmaker's equation: $\dfrac{1}{f} = \dfrac{n_2 - n_1}{n_1}\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$
- Thin lenses in contact: $\dfrac{1}{f_\mathrm{eff}} = \sum_i \dfrac{1}{f_i}$
- Two thin lenses separated by length L: $\dfrac{1}{f_\mathrm{eff}} = \dfrac{1}{f_1} + \dfrac{1}{f_2} - \dfrac{L}{f_1 f_2}$ (from principal plane 1)
- Relative aperture (i.e., f-#, f/stop): $\mathcal{R} = f/D$
- Resolving power: $\mathcal{R} \equiv \dfrac{\lambda}{(\Delta\lambda)_\mathrm{min}} = b\dfrac{dn_\lambda}{d\lambda}$
- Index of refraction of prism: $n = \dfrac{\sin((A+\delta)/2)}{\sin(A/2)}$
- Minimum deviation of a prism: $\delta \approx A(n-1)$
- Normal dispersive curve: $n_\lambda = A + \dfrac{B}{\lambda^2} + \dfrac{C}{\lambda^4} + ...,$ where $A, B, C, ...$ are empirical (Cauchy approximation).
- Dispersion: $\mathcal{D} = n_\lambda/\lambda$
- Dispersive power: $\Delta \equiv \dfrac{\mathcal{D}}{\delta} = \dfrac{n_F - n_C}{n_D - 1}$

Fraunhofer Lines			
		Glass n	
λ (nm)	Char.	Crown	Flint
486.1	F, blue	1.5286	1.7328
589.2	D, yellow	1.5230	1.7205
656.3	C, red	1.5205	1.7076

- Cartesian ovoid of revolution: constant $= n_\mathrm{o}(x^2+y^2)^{1/2} + n_\mathrm{i}[y^2 + (s_\mathrm{o} + s_\mathrm{i} - x)^2]^{1/2}$
- Newtonian equation for thin lens: $xx' = f^2$, where x is distance from object to focal point on object side and x' is distance from focal point on other side to image.
- Image length of cylindrical lens: $\overline{AB} = \left(\dfrac{s+s'}{s}\right)\overline{CL}$
- Thick lens equations:
  - ∗ $\dfrac{1}{f_1} = \dfrac{n_\mathrm{L} - n'}{nR_2} - \dfrac{n_\mathrm{L} - n}{nR_1} - \dfrac{(n_\mathrm{L} - n)(n_\mathrm{L} - n')}{nn_\mathrm{L}}\dfrac{t}{R_1 R_2}$
  - ∗ $f_2 = -n' f_1/n$
  - ∗ Principal point 1: $r = \dfrac{n_\mathrm{L} - n'}{n_\mathrm{L} R_2} f_1 t$
  - ∗ Principal point 2: $s = -\dfrac{n_\mathrm{L} - n}{n_\mathrm{L} R_1} f_2 t$
  - ∗ Nodal point 1: $v = \left(1 - \dfrac{n'}{n} + \dfrac{n_\mathrm{L} - n'}{n_\mathrm{L} R_2} t\right) f_1$
  - ∗ Nodal point 2: $w = \left(1 - \dfrac{n}{n'} - \dfrac{n_\mathrm{L} - n}{n_\mathrm{L} R_1} t\right) f_2$
  - ∗ Object-image distance; $-\dfrac{f_1}{s_\mathrm{o}} + \dfrac{f_2}{s_\mathrm{i}} = 1$
  - ∗ Lateral magnification: $m = -s_\mathrm{i}/s_\mathrm{o}$
- $\mathrm{Det}\,[\mathcal{M}_\mathrm{sys}] = \left|\begin{bmatrix} A & B \\ C & D \end{bmatrix}\right| = AD - BC = \dfrac{n_0}{n_f}$
- Simple ray-transfer matrices:
  - Translation: $\mathcal{T} = \begin{bmatrix} 1 & L \\ 0 & 1 \end{bmatrix}$
  - Refraction (spherical): $\mathcal{R}_\mathrm{sph} = \begin{bmatrix} 1 & 0 \\ \dfrac{n - n'}{n'R} & \dfrac{n}{n'} \end{bmatrix}$
  - Thin-lens: $\mathcal{L} = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$
  - Spherical mirror: $\mathcal{S} = \begin{bmatrix} 1 & 0 \\ \dfrac{2}{R} & 1 \end{bmatrix}$
- Cardinal point locations in terms of system matrix elements (with respect to (wrt) input (1) and output (2) reference planes unless noted):

F_1: $p = \dfrac{D}{C}$

$$f_1 = p - r = \frac{\frac{n_0}{n_f}}{C} \text{ (wrt prin. plane)}$$

F_2: $q = -\dfrac{A}{C}$

$$f_2 = q - s = -\frac{1}{C} \text{ (wrt prin. plane)}$$

H_1: $r = \dfrac{D - \frac{n_0}{n_f}}{C}$ $\quad$ H_2: $s = \dfrac{1 - A}{C}$

N_1: $v = \dfrac{D - 1}{C}$ $\quad$ N_2: $w = \dfrac{\frac{n_0}{n_f} - A}{C}$

- Wave-particle duality
  - Momentum of particle: $p = \dfrac{\sqrt{E^2 - (mc^2)^2}}{c}$
  - Wavelength of particle: $\lambda = h/p$
  - Speed of particle: $v = pc^2/E$
- Wave equations:
  - Generic 3D wave equation: $\psi(r,t) = Ae^{i(\vec{\mathbf{k}} \bullet \vec{\mathbf{r}} - \omega t)}$, where physical wave is real or imaginary component.
  - Wave number: $k = 2\pi/\lambda$
  - Angular frequency: $\omega = 2\pi\nu$
  - Harmonic condition: $\nabla^2\psi = \dfrac{1}{v^2}\dfrac{\partial^2\psi}{\partial t^2}$
- Superposition of $\mathcal{N}$ harmonic (plane) waves, $E_i = E_{0i}\cos(\alpha_i - \omega t)$:
  - Resulting constant phase:
    $$\tan\alpha = \frac{\sum_{i=1}^{\mathcal{N}} E_{0i}\sin\alpha_i}{\sum_{i=1}^{\mathcal{N}} E_{0i}\cos\alpha_i}$$
  - Resulting amplitude:
    $$E_0^2 = \sum_{i=1}^{\mathcal{N}} E_{0i}^2 + 2\sum_{i=1}^{\mathcal{N}}\sum_{j=i+1}^{\mathcal{N}} E_{0i}E_{0j}\cos(\alpha_j - \alpha_i)$$
- Electromagnetic (EM) waves:
  - Electric and magnetic field relation: $|\vec{\mathbf{E}}| = v|\vec{\mathbf{B}}|$, where v is speed of EM wave.
  - Total energy density of EM wave: $u = \varepsilon E^2 = B^2/\mu$
  - Poynting vector: $\vec{\mathbf{S}} = \dfrac{\varepsilon c^2}{n^2}\vec{\mathbf{E}} \times \vec{\mathbf{B}}$ (power per unit area of EM field).
  - Irradiance: $E_e = \left\langle |\vec{\mathbf{S}}| \right\rangle = \dfrac{1}{2}\varepsilon_0 c^2 E_0 B_0$
- Fourier series, where T is a period of $f(t)$:
  $$f(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty} a_m\cos(m\omega t) + \sum_{n=1}^{\infty} b_m\sin(m\omega t)$$
  * $a_0 = \dfrac{2}{T}\displaystyle\int_{t_0}^{t_0+T} f(t)dt$
  * $a_n = \dfrac{2}{T}\displaystyle\int_{t_0}^{t_0+T} f(t)\cos(n\omega t)dt$
  * $b_n = \dfrac{2}{T}\displaystyle\int_{t_0}^{t_0+T} f(t)\sin(n\omega t)dt$
- Fourier-transform pair:
  $$f(t) = \int_{-\infty}^{+\infty} g(\omega)e^{-i\omega t}d\omega \text{ and } g(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(t)e^{i\omega t}dt$$
- Complex numbers:
  - Generic complex number: $\tilde{z} = a + ib$ where
    * $a = \mathrm{Re}(\tilde{z})$,
    * $b = \mathrm{Im}(\tilde{z})$, and
    * $|\tilde{z}|^2 = a^2 + b^2$.
  - Euler's formula: $e^{i\theta} = \cos\theta + i\sin\theta$, where $\theta = \tan^{-1}\left(\dfrac{b}{a}\right)$ for $\tilde{z}$ above.
  - Complex conjugate: $\tilde{z}^* = a - ib = |\tilde{z}|e^{-i\theta}$
- Miscellaneous Physics:
  - Wien's Displacement Law: $\lambda_{\text{peak}} = \kappa T^{-1}$
  - Stefan-Boltzmann Law: $j = \sigma T^4$, where j is flux at surface.
  - Luminosity-flux relation: $L = A F$, where A is area.
  - Radiation pressure: $P = \dfrac{F}{c}$, where F is flux.
  - Ave. kinetic energy of particles: $E \approx k_{\mathrm{B}} T$
- Quadratic solution for $ax^2 + bx + c = 0$: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Laplacian operator: $\nabla^2 \equiv \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2} + \dfrac{\partial^2}{\partial z^2}$
- Scalar/dot product: $\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = AB\cos\theta$, where θ is angle between, or: $\vec{\mathbf{A}} \bullet \vec{\mathbf{B}} = \displaystyle\sum_{u \text{ comp.}} A_u B_u$
- Vector/cross product: $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = AB\sin\theta$, in direction $\perp$ to $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$, or: $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = \displaystyle\sum_{u \text{ comp.}}\sum_{w \text{ comp.}} A_u B_w(\hat{\mathbf{u}} \times \hat{\mathbf{w}})$
- Taylor series: $f(x) = \displaystyle\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n$, where $f^{(n)}(a)$ denotes nth derivative of f evaluated at point a. Factorial of zero is one (i.e., $0! = 1$).
- Useful trigonometric equations:
  * $\displaystyle\int \sin x dx = -\cos x$
  * $\displaystyle\int \cos x dx = \sin x$
  * $\sin(\theta \pm \phi) = \sin\theta\cos\phi \pm \cos\theta\sin\phi$
  * $\cos(\theta \pm \phi) = \cos\theta\cos\phi \mp \sin\theta\sin\phi$
  * $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$
  * $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$
  * $\sin\left(n\pi + \dfrac{\pi}{2}\right) = \cos n\pi = (-1)^n$ where integer $n \geq 0$