ASTR180: Principles of Astronomy I

Equations & Constants

Version 4: August 9, 2017

Common measurement units:

- **Length:**
  - $1\,\text{Å} = 10^{-10}\,\text{m}$
  - $1\,\text{AU} = 1.496 \times 10^{8}\,\text{km}$
  - $1\,\text{Ly} = 9.46 \times 10^{12}\,\text{km}$
  - $1\,\text{pc} = 3.09 \times 10^{13}\,\text{km}$
  - $1\,\text{R}_\text{MW}$ (Milky Way radius) $\approx 17\,\text{kpc}$
- **Mass:**
  - Atomic unit: $1\,\text{u} = 1.661 \times 10^{-27}\,\text{kg}$
    $= 931.5\ \text{MeV}\,c^{-2}$
  - Milky Way mass: $1\,\text{M}_\text{MW} \approx 1.25 \times 10^{12}\,\text{M}_\odot$

Particle	*Mass (*u*)*
Electron (e^-)	5.486×10^{-4}
Proton (p^+)	1.0073
Neutron (n^0)	1.0087
Hydrogen (^{1}H)	1.0079
Deuterium (D or ^{2}H)	2.0136
Helium-4 (^{4}He)	4.0015
Carbon-12 (^{12}C)	$12.\bar{0}$

- **Miscellaneous:**
  - Force: $1\,\text{N}$ (newton) $= 1\,\text{kg}\,\text{m}\,\text{s}^{-2}$
  - Pressure: $1\,\text{Pa}$ (pascal) $= 1\,\text{N}\,\text{m}^{-2}$
    $1\,\text{Pa} = 9.87{\times}10^{-6}\,\text{atm}$ (atmosphere)
  - Temperature: $X\,\text{K}$ - $273\,\text{K} = X^\circ\,\text{C}$, where X is any number
  - Energy:
    - Joule: $1\,\text{J} = 1\,\text{kg}\,\text{m}^2\,\text{s}^{-2}$
    - Electron-volt: $1\,\text{eV} = 1.602 \times 10^{-19}\,\text{J}$
  - Luminosity: $1\,\text{W}$ (watt) $= 1\,\text{J}\,\text{s}^{-1}$
  - Angular measurements:
    - Degree: $1^\circ = \dfrac{1}{360}$ of a circle
      $= \dfrac{2\pi}{360}\,\text{rad}$ (radian)
    - Arcminute ($'$): $1' = \dfrac{1}{60}\,^\circ$
    - Arcsecond ($''$): $1'' = \dfrac{1}{60}\,'$

Solar brightness:

- Total luminosity: $1\,\text{L}_\odot = 3.83 \times 10^{26}\,\text{W}$
- Apparent V-band magnitude: $m_{V,\odot} = -26.9\,\text{mag}$
- Absolute V-band magnitude: $M_{V,\odot} = 4.83\,\text{mag}$

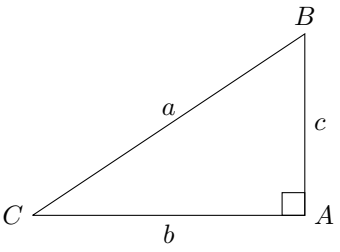
Constants:

- Speed of light (in vacuum): $c = 2.998 \times 10^{5}\,\text{km}\,\text{s}^{-1}$
- Gravitational constant:
  $G = 6.67 \times 10^{-11}\,\text{m}^3\,\text{kg}^{-1}\,\text{s}^{-2}$
- Acceleration of gravity at Earth's surface:
  $g = 9.81\,\text{m}\,\text{s}^{-2}$
- Wien's Displacement Law constant:
  $\kappa = 2.898 \times 10^{6}\,\text{nm}\,\text{K}$
- Stefan-Boltzmann constant:
  $\sigma = 5.67 \times 10^{-8}\,\text{W}\,\text{m}^{-2}\,\text{K}^{-4}$
- Planck constant: $h = 6.626 \times 10^{-34}\,\text{J}\,\text{s}$
- Boltzmann constant: $k_\text{B} = 1.381 \times 10^{-23}\,\text{J}\,\text{K}^{-1}$
- Coulomb constant: $k_\text{C} = 9 \times 10^{9}\,\text{N}\,\text{m}^2\,\text{C}^{-2}$. (The coulomb, C, is the fundamental unit of charge.)
- Hubble constant today: $H_0 = 71.9\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$
- Mathematical constant: $e \approx 2.71828$. (e is associated with natural logarithms, exponential decays, etc.)

Equations

- Velocity: $v = \dfrac{d}{t}$, where d is distance and t is time.
- Acceleration: $a = \dfrac{v}{t}$ (see previous).
- Wavelength-frequency relation: $c = \lambda\nu$,
  where c is speed of light in vacuum.
- Photon energy: $E = h\nu$,
  where h is Planck constant.
- Redshift: $z = \dfrac{\lambda_\text{obs} - \lambda_\text{em}}{\lambda_\text{em}} = \dfrac{v}{c}$.
- Hubble's Law: $v = H_0\,d$.
- Parallax: $d = \dfrac{1}{p}$, where parallax, p, is in arcseconds ($''$) to give distance in parsec (pc).
- Newton's Second Law (basic net force law): $F_\text{net} = ma$, where m is mass and a is acceleration. Weight is force of gravity.
- Linear momentum: $p = mv$, where m is mass and v is velocity.
- Angular momentum: $L = mv_\perp r$, where m is mass and $v_\perp$ is velocity perpendicular to r.
- Gravitational force: $F_\text{grav} = \dfrac{GMm}{r^2}$.
- Gravitational potential energy: $U = -\dfrac{GMm}{r}$
- Potential energy on Earth: $U = mgh$ where h is height above Earth's surface.
- Kinetic energy: $K = \dfrac{1}{2}mv^2$ where m is mass.
- Escape velocity: $v_\text{esc} = \sqrt{\dfrac{2GM}{r}}$.
- Electromagnetic or Coulomb force:
  $F_\text{EM} = \dfrac{k_\text{C}\,q_1\,q_2}{r^2}$, where $q_\#$ are charges.
- Newton's generalization of Kepler's 3rd Law:
  $p^2 = \dfrac{4\pi^2}{G(M+m)}\,a^3$,
  where p is orbital period and a is the total distance between masses M and m.
- Orbital Velocity Law: $M_\text{encl} = \dfrac{rv^2}{G}$, where at radius r, objects (in circular orbits) orbit with velocity v, and total mass enclosed by orbit is M_encl.
- Wien's Displacement Law: $\lambda_\text{peak} = \kappa\,T^{-1}$,
  where κ is the Wien's Displacement Law constant.
- Stefan-Boltzmann Law: $j = \sigma\,T^4$,
  where j is flux at surface, and σ is the Stefan-Boltzmann constant.
- Power (or luminosity if emitted): $P = \dfrac{E}{t}$.
- Luminosity-flux relation: $L = A\,F$, where A is area.
- Magnitude equation: $m_1 - m_2 = -2.5\log_{10}\left(\dfrac{F_1}{F_2}\right)$,
  where object #1 has magnitude m_1 and flux F_1 and object #2 has magnitude m_2 and flux F_2.

- Absolute magnitude equation:
  $m - M = -5 + 5 \log_{10} d$,
  where m is apparent magnitude, M is absolute magnitude of the same object, and d is distance in parsecs (pc).
- Mass-energy equivalence: $E = mc^2$.
- Pressure: $P = \frac{F}{A}$, where F is force and A is area.
- Radiation pressure: $P = \frac{F}{c}$, where F is flux.
- Average kinetic (i.e., motion) energy of particles: $E \approx k_{\rm B}\, T$, where $k_{\rm B}$ is the Boltzmann constant.
- Ideal gas law (gas pressure): $P = n\, k_{\rm B}\, T$, where n is number of particles per unit volume and $k_{\rm B}$ is the Boltzmann constant.
- Logarithm Rules, where b is the base (e.g. base-10 is $\log_{10}()$, or often $\log()$; natural logarithm is base e, so $\log_e()$, often $\ln()$):
  - $\log_b(xy) = \log_b x + \log_b y$
  - $\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$
  - $\log_b(x^y) = y \log_b x$
  - $b^{\log_b x} = x$
- Quadratic solution for $ax^2 + bx + c = 0$:
  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- Density: $\rho = \frac{m}{V}$ where m is mass and V is volume.
- Geometry:
  - Circumference of a circle: $d = 2\pi r$
  - Area of a circle: $A = \pi r^2$
  - Surface area of a sphere: $A = 4\pi r^2$
  - Volume of a sphere: $V = \frac{4}{3}\pi r^3$
  - Surface area of a cylinder: $A = 2\pi r h + 2\pi r^2$, where h is height
  - Volume of a cylinder: $V = \pi r^2 h$
- Trigonometry:
  - $a^2 = b^2 + c^2$
  - $A + B + C = 180°$
  - $\sin B = \cos C = \frac{b}{a}$
  - $\cos B = \sin C = \frac{c}{a}$
  - $\tan B = \frac{b}{c} = \frac{1}{\tan C}$



Common prefixes:

- Giga = 10^9 or billion; denoted as G.
- Mega = 10^6 or million; denoted as M.
- Kilo = 10^3 or thousand; denoted as k.
- Centi = 10^{-2} or one-hundredth; denoted as c.
- Milli = 10^{-3} or one-thousandth; denoted as m.
- Micro = 10^{-6} or one-millionth; denoted as μ and sometimes called *micron* when applied to meters.
- Nano = 10^{-9} or one-billionth; denoted as n.

Body	Radius (km)	Mass (kg)	Orbital Semimajor Axis (AU)	Orbital Period (yr)	Sidereal Rotation Period[a] (Earth days)
Sun	695,000	1.99×10^{30}	...	...	25.4
Mercury	2,440	3.30×10^{23}	0.387	0.2409	58.6
Venus	6,051	4.87×10^{24}	0.723	0.6152	−243.0
Earth	6,378	5.97×10^{24}	1.00	1.0	0.9973
Mars	3,397	6.42×10^{23}	1.524	1.881	1.026
Jupiter	71,492	1.90×10^{27}	5.203	11.86	0.41
Saturn	60,268	5.69×10^{26}	9.54	29.5	0.44
Uranus	25,559	8.66×10^{25}	19.19	84.01	−0.72
Neptune	24,764	1.03×10^{26}	30.06	164.8	0.67
Pluto[b]	1,160	1.31×10^{22}	39.48	248.0	−6.39
Eris[b]	1,430	1.66×10^{22}	67.67	557.	15.8

Satellite	Planet	Radius or Dimensions (km)	Distance from Planet (10^3 km)	Orbital Period[a] (Earth days)	Mass (kg)
Moon	Earth	1738	384.4	27.322	7.349×10^{22}
Phobos	Mars	$13 \times 11 \times 9$	9.38	0.319	1.3×10^{16}
Deimos	Mars	$8 \times 6 \times 5$	23.5	1.263	1.8×10^{15}
Io	Jupiter	1821	421.6	1.796	8.933×10^{22}
Europa	Jupiter	1565	670.9	3.551	4.797×10^{22}
Ganymede	Jupiter	2634	1070.0	7.155	1.482×10^{23}
Callisto	Jupiter	2043	1883.0	16.689	1.076×10^{23}
Titan	Saturn	2575	1221.85	15.945	1.35×10^{23}
Miranda	Uranus	236	129.8	1.413	6.6×10^{19}
Triton	Neptune	1352.6	354.59	−5.875	2.14×10^{22}
Charon	Pluto[b]	593	17.5	6.4	1.56×10^{21}
Dysnomia	Eris[b]	50	37.4	15.8	...

[a] Negative sign indicate rotation is backward relative to other planets or backward orbit (if satellite).

[b] Under the IAU definition of August 2006, Pluto and Eris are officially designated "dwarf planets."