

AY5, Summer Session I, 2008: Constants, Equations, Conversion Factors, Terms

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1. Common prefixes (<http://physics.nist.gov/cuu/Units/prefixes.html>)
 - (a) Giga = 10^9 or billion; denoted as G (*e.g.*, 13.7 billion years = 13.7 Gyr).
 - (b) Mega = 10^6 or million; denoted as M (*e.g.*, 100 million years = 100 Myr).
 - (c) Kilo = 10^3 or thousand; denoted as k (*e.g.*, 11 kiloparsec = 11 kpc).
 - (d) Centi = 10^{-2} or one-hundredth; denoted as c (*e.g.*, 3 centimeters = 3 cm)
 - (e) Milli = 10^{-3} or one-thousandth; denoted as m (*e.g.*, 5 millimeters = 5 mm)
 - (f) Micro = 10^{-6} or one-millionth; denoted as μ and sometimes called *micron* when modifying meters (*e.g.*, 24 micron = 24 μm).
 - (g) Nano = 10^{-9} or one-billionth; denoted as n (*e.g.*, 10 nanoseconds = 10 ns).

2. Common measurement units:
 - (a) Distance/length:
 - i. 1 AU (astronomical unit) = 1.496×10^8 km (average distance between Earth and Sun)
 - ii. 1 Å (Ångstrom) = 10 nm = 10^{-10} m
 - iii. 1 Ly (light-year) = 9.46×10^{12} km (distance light travels in a year in a vacuum)
 - iv. 1 pc (parsec) = 3.09×10^{13} km = 3.26 Ly
 - v. 1 R_{SUN} (or R_{\odot} ; solar radii) = 695,000 km
 - vi. 1 R_{EARTH} (or R_{\oplus} ; earth radii) = 6,378 km
 - vii. z (redshift) = $\frac{\lambda_{\text{observed}} - \lambda_{\text{rest}}}{\lambda_{\text{rest}}} = \frac{v}{c}$,
 where $\lambda_{\text{observed}}$ is the observed (measure) wavelength of the light from a source that is moving, λ_{rest} is the laboratory value of the wavelength of the light when the source is not moving, v is the velocity of the object, and c is the speed of light.
 - (b) Mass:
 - i. 1 M_{SUN} (or M_{\odot} ; solar masses) = 2×10^{30} kg
 - ii. 1 M_{EARTH} (or M_{\oplus} ; solar masses) = 5.97×10^{24} kg
 - iii. 1 u = 931.5 MeV/ c^2

Particle	Mass		
	(kg)	(u)	(MeV/ c^2)
Electron (e^-)	9.109×10^{-31}	5.486×10^{-4}	0.5109991
Proton (p^+)	1.672×10^{-27}	1.0073	938.2723
Neutron (n^0)	1.675×10^{-27}	1.0087	939.5656
Deuterium (D)	3.344×10^{-27}	2.0136	1875.6134
Helium-4 (${}^4\text{He}$)	6.645×10^{-27}	4.0015	3727.3803

- (c) Time:
 - i. 1 yr (year): not light-year

ii. z (cosmological redshift): the higher the redshift of an object, the farther away the object is in distance, the farther back in time the object is, and the longer the light has traveled in time and distance.

(d) Miscellaneous:

- i. c (speed of light) = 3×10^5 km/s
- ii. 1 K (Kelvin) = $1^\circ \text{C} + 273$
- iii. $1 L_{\text{SUN}}$ (or L_{\odot} ; solar luminosity) = 3.8×10^{26} watts (W), where 1 W = 1 joule/s (or J/s)
- iv. $1 \text{ J} = 1 \text{ kg m}^2 \text{ s}^{-2}$
- v. $1 \text{ eV} = 1.602 \times 10^{-19} \text{ J}$
- vi. $1 \text{ Pa} = 1 \text{ N/m}^2 = 9.87 \times 10^{-6} \text{ atm}$ (pressure)
- vii. $1 \text{ N} = 1 \text{ kg m s}^{-2}$

3. Useful constants and common symbols:

- (a) c (speed of light) = 3×10^5 km/s
- (b) Wien's Law constant $\kappa = 2.898 \times 10^6 \text{ m K}$
- (c) Stefan-Boltzmann constant $\sigma = 5.67 \times 10^{-8} \text{ W m}^{-2} \text{ K}^{-4}$
- (d) Gravitational constant $G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$
- (e) Planck constant $h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s}$
- (f) Boltzmann constant $k = 1.381 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$
- (g) Hubble constant $H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}$
- (h) Solar units
 - i. L_{SUN} (or L_{\odot} ; solar luminosity) = 3.8×10^{26} watts (W), where 1 W = 1 joule/s (or J/s)
 - ii. M_{SUN} (or M_{\odot} ; solar masses) = $2 \times 10^{30} \text{ kg}$
 - iii. R_{SUN} (or R_{\odot} ; solar radii) = 695,000 km

4. Equations

- (a) Wien's Law: $\lambda_{peak} = \kappa T^{-1}$
the wavelength of peak intensity of a blackbody.
- (b) Stefan-Boltzmann Law: $j = F = \sigma T^4$,
where in class j was "energy flux density" and F is the shortened "flux." Also applied to a blackbody.
- (c) Luminosity-flux relation: $L = A \cdot F$,
where A is area, in general, and $L = 4\pi R^2 F$ for sphere.
- (d) Wavelength-frequency relation $c = \lambda\nu$,
where ν is the frequency in s^{-1} (or cycles per second) and λ is the wavelength in meters to get c in meters per second.
- (e) Photon energy: $E = h\nu = hc/\lambda$
- (f) Particle energy: $E = kT$
- (g) Mass-energy equivalence: $E = mc^2$,
where m is mass and c is the speed of light.

- (h) Hubble's Law: $v = H_0 d$
- (i) Gravitational force: $F_{grav} = \frac{GMm}{r^2}$
- (j) Pressure: $P = F/A$,
where F is a force and A is the area.
- (k) Newton's version of Kepler's 3rd Law: $p^3 = \frac{4\pi^2}{G(M+m)} a^3$,
where p is the period of the orbit and a is the average distance between the two masses M and m .
- (l) Ideal gas law (gas pressure): $P = nkT$,
where n is the number of particles per unit volume, k is the Boltzmann constant, and T is temperature.
- (m) Radiation pressure: $P = F/c$,
where F is the flux.
- (n) Generic magnitude (not necessarily apparent magnitude): $m_1 - m_2 = -2.5 \log_{10} \left(\frac{F_1}{F_2} \right)$,
where m_1 is the magnitude of object 1 that has flux F_1 and m_2 is the magnitude of object 2 that has flux F_2 .
- (o) Absolute magnitude: $m - M = -5 + 5 \log_{10} d$,
where m is the apparent magnitude and M is the absolute magnitude of the same object and d is the distance from the observer to the object.

5. Terms and concepts

- Absorption (absorb): process whereby radiation is removed from an impinging radiation source by electrons as they move to higher-energy states in the atoms.
- Abundance: the total number of atoms of an element in an object.
- Blackbody (blackbody radiation): idealized model whereby the radiation emitted is strictly due to the object's temperature.
- Brightness: typically an ill-defined term, used loosely to indicate luminosity or flux.
- "Burning": an imprecise term. Astronomers typically mean nuclear fusion and not the chemical sense (whereby oxygen is involved).
- Center of mass: the point about which two gravitationally-bound masses orbit. The point is closer to the more massive object.
- Color: a specific wavelength of visible electromagnetic radiation. Typically, when used in reference to astronomical object, it indicates the wavelength at which the object radiates the majority of its visible light.
- Conservation (conserve): a physical quantity cannot be created or destroyed, simply converted.
 - (a) Conservation of energy: energy and matter (since $E = mc^2$). For example, in the main-sequence stellar core, four particles of (ionized) hydrogen are fused into (ionized) helium in the stellar core. (I specify ionized because there are no electrons attached to the elements; ionized hydrogen is just a proton.) The specific reaction is written thus:



where ΔE is the energy *released* by the reaction. (If ΔE had been on the left-hand side of the above reaction, it would have indicated that there was energy *added* to the reaction in order to make it proceed.)

Applying the mass-energy equivalence (Einstein's equation), total mass-energy at the beginning of the reaction is:

$$E_{initial} = 4 m_p c^2,$$

where m_p is the mass of the proton (or ionized hydrogen). The total mass-energy at the end of the reaction is:

$$E_{final} = m_{He-4} c^2 + \Delta E,$$

where m_{He-4} is the mass of the (ionized) ${}^4\text{He}$ atom. (The notation, ${}^4\text{He}$, He-4, and helium-4, indicates that the element is helium, He, and that the total number of protons and neutrons in the nucleus is four.)

By the law of conservation of energy, $E_{initial} = E_{final}$. With this fact, we can compute the value of the released energy ΔE :

$$4 m_p c^2 = m_{He-4} c^2 + \Delta E$$

$$\Delta E = 4 m_p c^2 - m_{He-4} c^2.$$

- Continuum (continuous): radiation produced by particles colliding and emitting energy covering the full range of wavelengths, as in the spectrum of a blackbody source.
- Cosmic microwave background radiation (CMBR): signature of the last-scattering surface after the Big Bang. It's a 3000 K blackbody at $z \approx 1000$. Today ($z = 0$) the CMBR is 2.73 K with its peak wavelength in the microwave range.
- Cosmology: the study of the formation, evolution, and large-scale structure of the Universe.
- Dimensional analysis: generally refers to the process of using units to assess the use of an equation. In this class, it is applied by the use of conversion factors to change units so they cancel and survive appropriately.
- Dust: particles of matter larger than a molecule and smaller than a pebble that exist in the Universe and affect the propagation of radiation. Typical terms describing the effects of dust are *reddening*, *extinguishing*, and *scattering*.
- Emission (emit): radiation produced by an electron in an atom moving to a lower-energy state.
- Epoch, era: period of time characterized by some event(s). For example, in the early Universe, there was an inflationary era or epoch.
- Flux: the amount of energy released per second per unit area.
- Fusion (nuclear): the process of combining nucleons (protons, neutrons) and/or even nuclei of elements into more tightly bound elements.
- Inquiry: learning science as science is done.
- Intensity: measure of the time-averaged energy flux.
- Light: sloppy term for electromagnetic radiation.

- Luminosity: the energy released per second.
- Magnitude: astronomical system of measuring the relative flux or luminosity of objects.
- Metal: in astronomy, any element heavier than helium.
- Metallicity: the ratio of the number of atoms of metals to the number of hydrogen atoms.
- Nucleus (nuclei): the positively charged core of an atom(s), comprised of protons and neutrons (nucleons) and held together by the strong nuclear force.
- Photon: the particle view of an electromagnetic wave.
- Projection: the two-dimensional rendering of a three-dimensional object.
- (Electromagnetic) radiation: the full range of energies carried by photons, which are also waves.
- Redshift (z): measure of the change in wavelength relative to the original wavelength due to the motion of an object through space (Doppler redshift) or of an object moving with spacetime (cosmological redshift).
- Reflection (reflect): the process of changing a photon's direction without changing its energy (much).
- Scale (verb and noun): (v) to relate physical values to more intuitive values; (n) essentially, the conversion factor that relates the two values.
- Scientific notation: formatting numbers so that there is one non-zero number to the left of the decimal place and any necessary numbers to the right of the decimal place and making up the difference in values by using powers of ten.
- Scientific process: the cyclical method of hypothesizing, experimenting, collaborating, (re-)evaluating and revising (repeat as necessary).
- Spectroscopy: breaking up radiation to measure the flux or luminosity at every wavelength.
- Spectrum (spectra): one-dimensional (*e.g.*, plot flux versus wavelength) or two-dimensional (*e.g.*, rainbow) rendering of flux or luminosity of radiation at all wavelengths.
- Structure: the physical organization of an object (*e.g.*, the interior of the Sun).
- z (redshift): cosmological redshift is the increase in the observed wavelength of light compared to the intrinsic wavelength due to the expansion of the Universe.