

AY 5, Summer Session I, 2008: Math Practice and Scale Models

Due in class, Monday, July 7th

Draft: June 22, 2008

Name:

Name(s) of collaborator(s):

Please feel free to ask for hints and/or clarification. Group work is strongly encouraged, but please identify who your collaborators are; each group member must submit the assignment copied in his/her own writing. Please put circles around final numerical answers. Homework must be legible, neat, and stapled. **Show your work**; the more work you show the more points you can accumulate even if the final answer is incorrect.

1. **Optional:** Practice using powers of ten and scientific notation. Remember that scientific notation conventionally has one digit to the left of the decimal point; this is the number that multiplies the powers of ten. Examples: $250 = 2.5 \times 10^2$, $0.25 = 2.5 \times 10^{-1}$.

(a) Express these numbers in scientific notation:

- i. 9999
- ii. 32.254
- iii. 0.10
- iv. 0.0000782

(b) Do the following calculations. *Hint:* write all numbers in scientific notation first, then do the arithmetic on the parts out front and re-express that with scientific notation. Then combine all the powers of ten last.

- i. $(2.5 \times 10^2) \times 4$
- ii. $(2.5 \times 10^2)/4$
- iii. $\frac{300 \times 2.5 \times 10^{33}}{7 \times 10^2 \times 1500 \times 10^{-22}}$
- iv. $1/250.0$

Note: Problem adapted from Prof. Sandy Faber's AY5 problem set.

2. **(8 pt total)** "Showing the units of a problem as you solve it usually makes the work much easier and also provides a useful way of checking your work. If an answer does not come out with the units you expect, you probably did something wrong. In general, working with units is very similar to working with numbers.

Before you begin any problem, think ahead and identify the units you expect for the final answer. Then operate on the units along with the numbers as you solve the problem. The following five guidelines may be helpful when you are working with units:

- Mathematically, it doesn't matter whether a unit is singular (*e.g.*, meter) or plural (*e.g.*, meters); we can use the same abbreviation (*e.g.*, m) for both.
- You cannot add or subtract numbers unless they have the *same* units. For example, 5 apples + 3 apples = 8 apples but the expression 5 apples + 3 oranges cannot be simplified further.
- You can multiply units, divide units, or raise units to powers. Look for key words that tell you what to do.
- Often the number you are given is not in the units you wish to work with. For example, you may be given that the speed of light is 300,000 km/s but need it in units of m/s for a particular problem. To convert the units, simply multiply the given number by a *conversion factor*: a fraction in which the numerator (top of the fraction) and denominator (bottom of the fraction) are equal, so that the value of the fraction is 1; the number in the denominator must have the units that you wish to change. For example, using the fact that 1 km/s = 10³ m/s:

$$\underbrace{300,000 \frac{\text{km}}{\text{s}}}_{\text{speed of light in km/s}} \times \underbrace{\frac{10^3 \text{ m}}{1 \text{ km}}}_{\text{conversion from km to m}} = \underbrace{3 \times 10^8 \frac{\text{m}}{\text{s}}}_{\text{speed of light in m/s}}$$

- It's easier to work with units if you replace division with multiplication by the reciprocal. For example, suppose you want to know how many minutes are represented by 300 seconds. We can find the answer by dividing 300 seconds by 60 seconds per minute:

$$300 \text{ s} \div 60 \frac{\text{s}}{\text{min}}$$

However it is easier to see the unit cancellations if we rewrite this expression by replacing the division with multiplication by the reciprocal:

$$300 \text{ s} \div 60 \frac{\text{s}}{\text{min}} = 300 \cancel{\text{s}} \times \underbrace{\frac{1 \text{ min}}{60 \cancel{\text{s}}}}_{\substack{\text{invert} \\ \text{and multiply}}} = 5 \text{ min.}''$$

Now on to the actual questions. (Remember to show your work.)

- How many seconds are there in one year? (**1 pt.**)
- Often astronomers use units of measurement that give an intuitive sense of the magnitude of the quantity. For example, the average distance between the Earth and the Sun is 1.496×10^8 km. This value is called the astronomical unit: $1 \text{ AU} = 1.496 \times 10^8$ km. Currently, extrasolar planets (*i.e.*, planets found orbiting distant stars) are typically measured to be about 0.5 AU from their parent stars. By using units of AU, astronomers can visualize these extrasolar planetary systems as smaller than our Earth's average orbit. **Question:** How many miles are in 1 AU given that $1 \text{ mi} = 1,609 \text{ m}$? (**1 pt.**)
- Another common unit in astronomy is *solar masses*, denoted M_{SUN} (or sometimes, M_{\odot}): $1 M_{\text{SUN}} = 2 \times 10^{30} \text{ kg}$. **Question:** How many solar masses is the Earth given that the mass of the Earth is $1 M_{\oplus} = 5.97 \times 10^{24} \text{ kg}$? (**1 pt.**)
- Convert the gravitational constant $G = 6.67 \times 10^{-11} \frac{\text{m}^3}{\text{kg s}^2}$ to units of $\frac{\text{AU}^3}{M_{\text{SUN}} \text{ yr}^2}$. (**5 pt.**)

- (e) **Extra credit:** Kepler’s third law of planetary motion “describes how a planet’s *orbital period* (the time it takes to complete one orbit of the Sun), measured in years, is related to its average distance from the Sun in astronomical units [...]”

$$(\text{orbital period in years})^2 = (\text{average distance in AU})^3.$$

The German astronomer Johannes Kepler determined this law *empirically* (*i.e.*, strictly through observations) in 1618. In 1687, Sir Isaac Newton developed his universal law of gravitation. With Newton’s law, one can generalize Kepler’s third law to any two orbiting bodies:

$$p^2 = \frac{4\pi^2}{G \times (M_1 + M_2)} a^3,$$

where p is the orbital period in seconds; G is the gravitational constant described previously; M_1 and M_2 are the masses of the two objects in kilograms; and a is the average distance between the two objects. **Question:** prove that Newton’s law reduces to Kepler’s law when the appropriate units are used. (5 pt.)

Note: Excerpts adapted from *The Cosmic Perspective, 2nd Ed.*

3. (18 pt total) Compute the scale model values of the sizes of the planets and the distances between objects in the solar system given that the Sun is to be represented by a water-polo ball with a radius of 4 in.

Name	Radius (km)	Distance from Sun ¹	
		(AU)	(10 ⁶ km)
Sun	695,000
Mercury	2,440	0.387	57.9
Venus	6,051	0.723	108.2
Earth	6,378	1.00	149.6
Mars	3,397	1.524	227.9
Jupiter	71,492	5.203	778.3
Saturn	60,268	9.539	1,427
Uranus	25,559	19.19	2,870
Neptune	24,764	30.06	4,497
Pluto ²	1,160	39.54	5,916

- (a) Reproduce the above table but with ‘Radius’ in inches (the Sun being 4 in) and the ‘Distance from Sun’ in feet (1 ft = 0.3048 m). You do not have to show every conversion but simply the *conversion factor* you use (see #2 above). Include two additional columns for the next parts. (3 pt.)
- (b) For the fourth column of your table, list a common object about the same size as the scaled ‘planet’ (*e.g.*, the Sun is a water-polo ball). (5 pt.)
- (c) For the fifth column of your table, describe where each planet would be in this scale model if you place the Sun/water-polo ball on the wall between Nat. Sci. 2 Annex and Thimann Lecture Hall and face west (towards Kresge College). (10 pt.)

¹Semi-major axis of orbit

²Ask me about a conspiracy theory about Pluto and its demoted status.

- (d) **Extra credit:** The Sun's nearest stellar neighbor is Proxima Centauri, which is 4.2 light-years away ($1 \text{ ly} = 9.46 \times 10^{12} \text{ km}$). In the scale model we've developed, name a location approximately as far away from our water-polo ball 'Sun' as the real Proxima Centauri is from our real Sun. (**2 pt.**)

Note: Solar system data from *The Cosmic Perspective, 2nd Ed.*

4. (**40 pt total**) Scale the U.S. government's budget to the size of a typical household budget. Use the actual or proposed federal budget from fiscal year 2007 2008, or 2009. Clearly state the assumed household budget (for example, the median Santa Cruz county annual household income is \$52,031³; the US median annual household income is \$48,201⁴; a UCSC astronomy graduate student earns \$25,604 per year.⁵)

Cite all sources consulted.

For an introduction to the U.S. government's budget (and some useful links), read the following tutorial by the American Association for the Advancement of Science (AAAS):

<http://www.aaas.org/spp/rd/budsem01.htm>.

- (a) Make a **five**-column table of U.S. government spending in (2) true dollars, (3) in scaled 'household income' dollars, and (4) in percentage of the budget. (Column #1 is the row label.) Do this for the following broad categories and the sub-categories in parentheses (8 rows total; **35 pt**):

- Mandatory/entitlement spending (Medicare)
- Military/security discretionary spending (Defense)
- Non-military/non-security discretionary spending (National Science Foundation or National Aeronautics and Space Administration)
- Supplemental and emergency spending (Wars in Iraq and Afghanistan or Hurricane Katrina relief)
- One category/sub-category of your choosing.

- (b) In the fifth column, name something the household could buy with the money spent in each of the chosen sub-categories. In other words, what common expense is approximately the cost of 'Medicare'? Is it a burrito, a car, a house, ...? (**5 pt.**)

Note: Yes, this is a little political of me.

³<http://quickfacts.census.gov/qfd/states/06/06087.html>

⁴http://en.wikipedia.org/wiki/Household_income_in_the_United_States

⁵Private communication. ;)